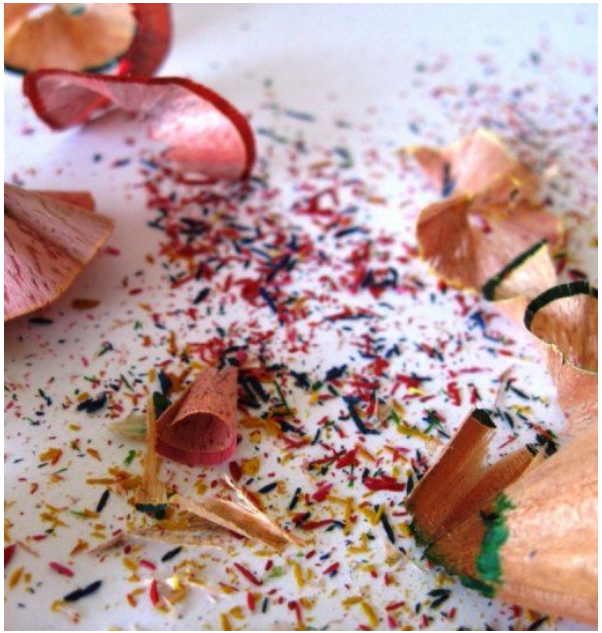
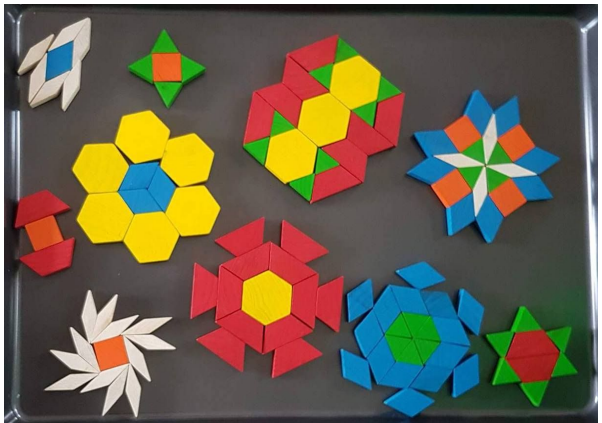
“Hard fun!” Learning complex subject matter and how to identify levels of skilled performance.





Steven K. Khan

skkhan@ualberta.ca

Assistant Professor

Mathematics Education & Computational Thinking

Department of Elementary Education, University of Alberta.

Learning Goals for FoTL Session	Learning Materials	Learner Actions	Learning Products
Participants will learn and be able to:	*Through engagement with:*	*This will involve*	*Learner will produce/dem*
DEFINE & RELATE Learning Trajectory, Variation Theory and Productive Struggle to learning of complex subject matter and skilled performance (teaching mathematics to diverse elementary-aged learners)	Game of Squares Puzzle. Skyscraper Puzzle. Perimeter trains problem. Finger counting problem. Student reflections (Mathcurations) Student comments.		
CONSTRUCT a learning trajectory for at least one mathematical game/puzzle		Play game. Analyse learning opportunities. Sequence.	Positive learning experience. A learning sequence.
DISCERN how this framework contributes to student (pre-service teacher) learning;			
MAKE CONNECTIONS/DRAW ANALOGIES with teaching and learning in other domains involving a combination of complex subject matter and skilled performance.	Questions and Discussion.		

Variation Theory

→ Introduces idea of Critical Features.

→ Helps students become aware of Dimensions that can Vary.

→ Used to help them discern:

- Variations in task purposes/learning goals
- Variations in instructional strategies.
- Way mathematical concepts are organized/attended to by the Professional Discourse Community.
- Helps to identify that there can be variation in skilled performance.

Learning Trajectories

→ Introduces idea that learning is developmental and facilitated by meaningful tasks effectively facilitated.

→ Gives a framework for mathematical concept development (learn complex subject matter)

→ Helps in identifying LEVELS OF SKILLED PERFORMANCE.

- Suggests relevant tasks to help move from one level to another.

Game Mechanics

→ Are one way to structure and sequence learning experiences for productive struggle.

→ Productive struggle does not have to be isolated/individual struggle.

→ Systems are structured but can be hacked/gamed - designed to be 'played'.

Productive Struggle

→ Introduces idea that effort & persistence are required to learn complex subject matter.
(learn complex subject matter)

Issue: Learning {Mathematics} is ‘hard’ (complex subject matter):

Things we know...

- **Effort, persistence, failures >> successes are NECESSARY for learning**
  - Learners need to appreciate the value of effort, struggle and failure, persistence in learning mathematics.
    - Growth Mindsets & Productive Struggle
- **Mathematical ideas are understood in particular ways in the Prof. Discourse Communities.**
  - Learners need to discern those particular ways (discernment is not ‘discovery’).
    - Variation Theory (Critical Features)
- **Initial successes should precede situations where failure is likely but not consequential. ‘Failures’ should present opportunities to learn and revise.**
    - Game Mechanics, Constructionism.
- **Conceptual Understanding gets more sophisticated/complex/connected over time.**
  - Learners need to experience a sensible/appropriate developmental sequence with learning tasks and experiences and reflection/critical connection & curation of experiences.
    - Learning Trajectories, Variation Theory.

Issue: Learning to teach Elementary Mathematics is ‘hard’ (complex subject matter)

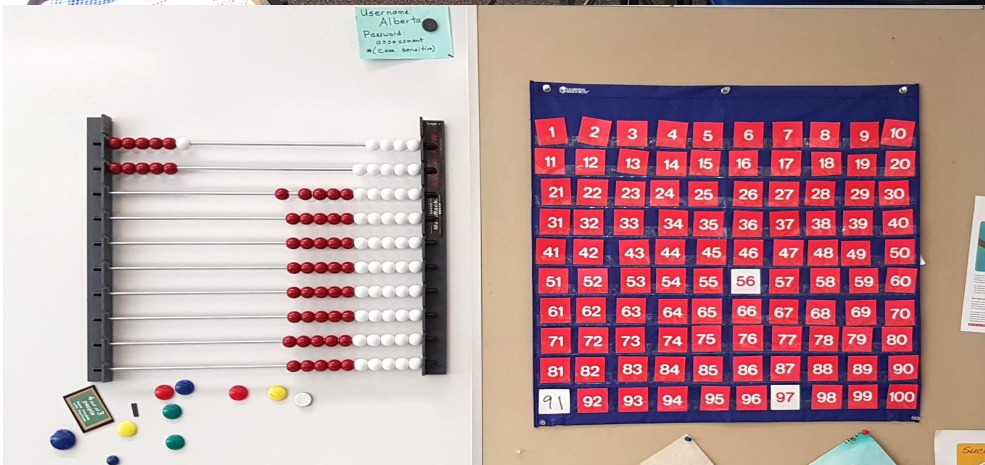
Things we know...

- **effort is necessary**
  - Growth Mindset
- **understanding early learning, discerning aspects of conceptual development are necessary**
  - Variation Theory, Learning Trajectories,
- **sensibly selecting and sequencing of appropriate (motivating) tasks-fit-to-purpose, and learner(s), and context are necessary**
  - Variation Theory, Learning Trajectories, Game Mechanics,
- **demands of tasks should increase but not elicit quitting.**
  - Productive Struggle
- **Learning to identifying different levels of skilled performance is necessary**
  - Learning Trajectories, Variation Theory
- **Knowledge of Mathematics is necessary (CK)**
- **Knowledge of effective instructional strategies fit-to-purpose is necessary**
- **Knowledge of Curriculum is required.**
- PLUS all the other stuff: designing, planning, monitoring, reporting, establishing and maintaining relationships, classroom management, inclusion, cultural relevancy etc.)

EDEL 316: Learning & Teaching Context

- Only **required** math pedagogy (‘methods’) course for K-6 Pre-Service Teachers in Elementary Mathematics.
- Focus is on Learning to Teach Mathematics to Elementary aged learners NOT Math Content Knowledge.
- Winter 2018 face-to-face sections with ~32 participants each.
- 8 weeks.
- 2 x 110 minutes/week/section.
- A proportion of students enter the course with anxiety about their mathematical proficiency and fears of teaching mathematics to young learners as well as challenges with mental arithmetic.
- Personal observation and engagement with elementary classroom teachers and pre-service teachers across 3 provinces in Canada suggests that EPSTs **don’t discern** different purposes for mathematical tasks (and consequently different TYPES/AFFORDANCES of mathematical tasks for learning) and **don’t discern** different levels skilled mathematical performance.

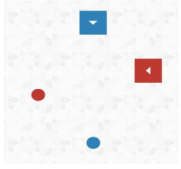
Classroom Physical Context

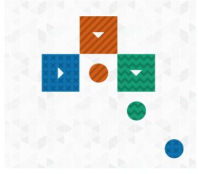


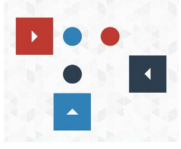
Game About Squares Puzzles

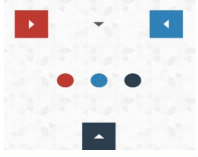
G.A.S.P.s

http://bit.ly/2BJxWMt









Learning Goals	Learning Materials	Learning Products	Success Criteria
Student-teachers will learn and be able to:	*This will involve*	*Learner will produce/dem*	*Learners and I will know they have successfully learned if...*
SOLVE G.A.S.Ps individually and collaboratively.	Game about squares app.	Screenshot of highest level achieved..	Can move up levels.
DEVELOP **FOUNDATIONAL EXPERIENTIAL UNDERSTANDING** of core course concepts such as *Learning Goals, Learning Trajectories, Critical Features, Success Criteria, Math Processes, Elements of Math Proficiency, Effective Teaching Practices* through REASONING by ANALOGY and REFLECTION using the GASPs.	Analogical reasoning. Reflection. Spreadsheet Slideshow Class discussion.	Reflective annotations in portfolio.	Can use the experience of solving GASPs to exemplify or relate core course concepts such as *Learning Goals, Learning Trajectories, Critical Features, Success Criteria, Math Processes, Elements of Math Proficiency, Effective Teaching Practices.*
EXPERIENCE guided (skill) development and RELATE to guided concept development.	Game about squares.		

Creating a Learning Trajectory for GASPs

- The intention is to use the experience of playing the game to provide a safe but challenging (non-mathematical) context for you to be able to return to relate ideas such as *Learning Goals, Learning Trajectories, Critical Features, Success Criteria, Math Processes, Elements of Math Proficiency, and Effective Teaching Practices.*
- The information below will only make sense IF you have played the game.
- Some of the concepts may not make full sense immediately depending on the specific learning sequence you have taken to get here.
  - As such you may need to review material covered earlier or come back to this lesson after you have encountered the relevant concepts and had additional experiences.

Play Game About Squares Here

http://gameaboutsquares.com

Learning Trajectories (Clements & Sarama, 2010)

- Children follow developmental progressions in learning and development.

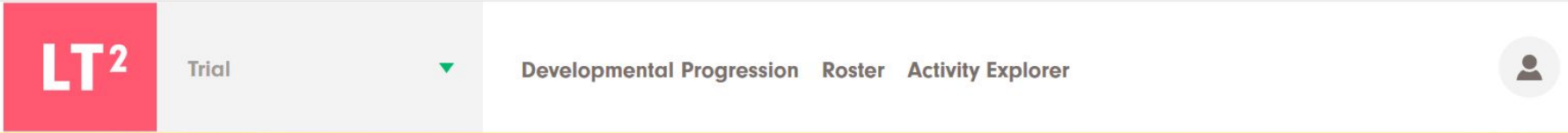
- Learning trajectories have three parts:

  a) a mathematical **GOAL**;

  b) a **DEVELOPMENTAL PATH** along which children develop to reach that goal; and

  c) a set of instructional activities, or **TASKS**, matched to each of the levels of thinking in that path that help children develop higher levels of thinking

- several learning trajectories are based on considerable research, eg. counting and arithmetic.
- others, such as patterning and measurement, have a smaller research base.
- few guidelines for more sophisticated math topics for older learners.



Learning and Teaching with *Learning Trajectories* (LT^2)

LT^2 is a web-based tool for early childhood educators to learn about how children think and learn about mathematics and how to teach mathematics to young children (birth to age 8). The website provides teachers with access to information about *Learning Trajectories* for math. Teachers can also review short video clips of children's thinking along the math *Learning Trajectories*. Users can access hundreds of classroom activity ideas to support children's development along the math trajectories.

Watch the video below to learn more! Then review our LT2 User Instructions to learn more about how to set up your account and use the site.

Log In or Register

https://www.learningtrajectories.org/

Learning Trajectory Subitizing

Overall Description: Children learn to recognize and name how many objects are in a small collection quickly, without counting ("subitize" comes from the Latin "to arrive suddenly").

Foundations	Namer of Small collections	Perceptual Subitizer to 4	Conceptual Subitizer to 5	Conceptual Subitizer with place value and skip counting.
Within the first year of life, is sensitive to number, but does not have the explicit knowledge of number.	Names groups of 1 to 2, sometimes 3.	Instantly recognizes collections up to 4 briefly shown and verbally names the number of items. Makes a small collection (1-4) with the same number as another collection.	Verbally labels different arrangements of numbers to about 5, when only shown briefly.	Verbally labels structured arrangements, shown only briefly, using groups, skip counting, and place value.
	Maker of Small Collections	**Perceptual Subitizer to 5**	**Conceptual Subitizer to 10**	**Conceptual Subitizer with place value and multiplication**
	Makes a small collection (usually 1-3) with the same number as another collection (not necessarily by matching). May name the number.	Instantly recognizes briefly shown collections up to 5 and verbally names the number of items. Recognizes and uses spatial and numeric structures beyond the situations in which they were already experienced (i.e., in which they were initially learned).	Verbally labels most briefly shown arrangements to 6, then up to 10, using groups.	Verbally labels structured arrangements shown only briefly, using groups, multiplication, and place value.
			Conceptual Subitizer to 20	
			Verbally labels structured arrangements up to 20, shown only briefly, using groups. Spontaneously makes use of a top-down strategy to subitizing larger qualities	

Play

Household Items for at Home Math
Play

Math at Home Tips and Activities
Play

Routines

Math in Everyday Routines
Routines

Math Talk
Routines

Small Group

Number Senser-Book with Groups of Number Quantities
Small Group

The Hungry Catepillar Book
Small Group

Small Group

Concentration
Small Group

Whole Group

Get the Number
Whole Group

Computer Activity

Subitize! Planets: Perceptual Subitizer to 5
Computer Activity

Small Group

Fantastic Five (Tricky Two)
Small Group

Whole Group

The Subitizing Song
Whole Group

Computer Activity

Concentration: Match Sums
Computer Activity

Small Group

Subitize Dots to
Small Group

Computer Activity

Subitize! Planets: Conceptual Subitizer Place Value
Computer Activity

Small Group

Snapshots
Small Group

Computer Activity

Subitize! Planets: Conceptual Subitizer Place Value
Computer Activity

Small Group

Subitize Dots
Small Group

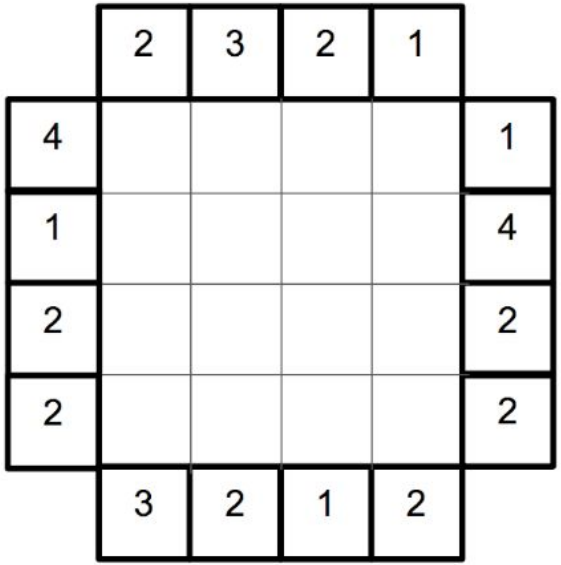
LT LEVEL	Foundational Block Clicker	Coloured Block Clicker	Clicker-Sequencer	Clicker-Sequencer Pusher	Proficient CSP
LT-LEVEL DESCRIPTION	Can click on block to make it move.	Clicks on more than one block.	Sequences moves of blocks.	Uses one block to move another block into place.	Combines sequencing and pushing.
CRITICAL FEATURES / DISCERNMENTS	Clicking on a square moves it one square unit in the direction that its arrow points	Clicking on a square moves it one square unit in the direction that its arrow points	The order in which the player moves the squares matters.	*S*quares can be used to pushed another square in the direction that the pushing square is pointing.	The structure of the setup matters.
SUCCESS CRITERIA	I can move a coloured square on top of a circle of the same colour.	I can move coloured squares on top of circles of the same colour.	I can sequence moves to move coloured squares on top of circles of the same colour.	I can use squares to move other squares and sequence moves to move coloured squares on top of circles of the same colour.	I can use squares to move other squares and sequence moves to move coloured squares on top of circles of the same colour.
PROCESS	Experimenting	Reasoning	Problem-Solving	PS, Connecting	E,R,PS, Cn, Com, Vis
PROFICIENCY	Procedural Fluency	Procedural Fluency	PF, Conceptual Understanding	PF, CU, Strategic Competence, Adaptive Reasoning.	PF, CS, SC, AR, Productive Disposition

LT LEVEL	Foundational Direction Changer	Developing CSP-DC	Proficient CSP-DC	EXPERT CSP-DC
LT-LEVEL DESCRIPTION	Uses directional arrows to change direction of block.	Sequences moves of blocks and uses directional arrows to change direction of blocks.	Sequences moves of blocks and uses directional arrows to change direction of blocks..	
CRITICAL FEATURES / DISCERNMENTS	Triangles change the direction a player can push a square.			
SUCCESS CRITERIA	I can use triangles to change the direction my square can move.			
PROCESS	Experimenting	Reasoning	PS, Com, Vis	PS, Com, Vis
PROFICIENCY	Procedural Fluency	PF, Conceptual Understanding.	PF, CS, SC, AR, PD	PF, CS, SC, AR, PD

Skyscraper Puzzles

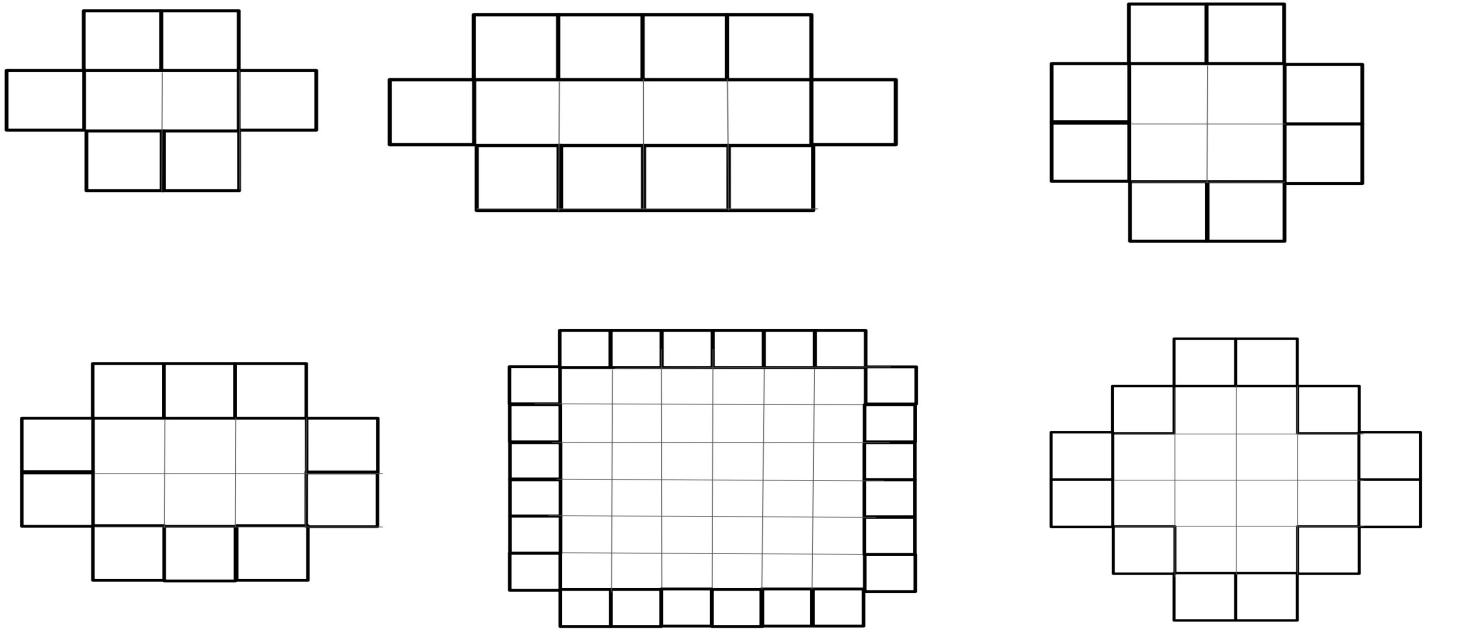
The Rules

- Using connecting cubes / cube-a-links build towers 1, 2, 3 and 4 cubes tall in each of the squares provided.
- Each row has skyscrapers of different heights (1 through 4), no duplicate sizes
- Each column has skyscrapers of different heights (1 through 4), no duplicate sizes
- The numbers on the outside (in grey) tell you how many skyscrapers you can see from that direction.
- Taller skyscrapers block your view of shorter ones



	2	3	2	1	
4					1
1					4
2					2
2					2
	3	2	1	2	

Learning to Work Backwards and Extend Beyond in order to scaffold concept / skill development instead of problem solving (this is hard for novices)**.**



- Place the tower inside a single square. Write how many towers you see from each side.
- Put a tower of different height in single square. Write how many towers see from each side.
- Move on to two towers.

PSTCs need to discern what can vary and think about the task purpose (learning intention)

Inquiry

If we allowed repeats could we determine uniquely the arrangement from the clues?
How might we have to change the clues?

What about if the board was not a rectangle, eg. cross? How might we have to change the rules?

Which clues give the most information? What are the fewest number of clues needed to build a puzzle?
Could you have a four on each side?

Does every completed Sudoku puzzle determine a skyscraper puzzle?
If yes, does it do so uniquely?i.e. Can one puzzle give rise to more than one skyscraper puzzle?

Learning Trajectory & Curriculum Connections

LT LEVEL	Fraction Basics	Fractions < or = to 1	Mixed & Improper Fractions (Separated)	Mixed & Improper Fractions (Together)	Operations & Mixed + Improper Fractions
LT-LEVEL DESCRIPTION	Can communicate that fractions are a part of a whole and communicate the parts of the fraction and what they represent	Understands and can represent fractions less or equal to one concretely, pictorially, and symbolically.	Can recognize and represent mixed and improper fractions.	Can relate improper fractions and mixed fractions with each other	Incorporates multiplication and division in understanding the relation between mixed and improper fractions.
CRITICAL FEATURES / DISCERNMENTS	Fractions are a part of a whole. Identify numerator as the number on the top and denominator as number on the bottom.	Representing fractions less than or equal to one through concrete manipulatives, pictorials, and symbols.	Recognition that mixed and improper fractions are fractions that are more than 1 and look different from proper fractions that are less than 1. Representing fractions that go beyond one through concrete manipulatives, pictorials, and symbols	Mixed and improper fractions appear different but also share similarities; therefore, one can find the equivalent mixed fraction of an improper fraction and an equivalent improper fraction of a mixed fraction	Multiplication and division help find the equivalent fraction (improper/mixed) of the given fraction (mixed/improper)
SUCCESS CRITERIA	I can communicate the characteristics of fractions. I can identify the numerator and denominator of fractions.	I can use concrete, pictorial, and symbolic forms to represent fractions equal to or less than one.	I can identify mixed and improper fractions and can represent these fractions concretely, pictorially, and symbolically.	I can understand the differences and similarities between improper and mixed fractions and use this knowledge to find the equivalent mixed/improper fraction of an/a improper/mixed fraction.	I can use multiplication on mixed fractions to find equivalent improper fractions, and I can use division on improper fractions to find equivalent mixed fractions
PROCESS(ES)	Communication	Communication, Visualization	Communication, visualization	Communication, Visualization	Communication, Connections, MME, Technology, Visualization
OVERALL CURRICULUM EXPECTATION	GO: Develop Number Sense	GO: Develop Number Sense	GO: Develop Number Sense	GO: Develop Number Sense	GO: Develop Number Sense
APPROPRIATE SPECIFIC CURRICULUM OBJECTIVE				SO: Relate improper fractions to mixed numbers and mixed numbers to improper fractions	

Learning Trajectory & Curriculum Connections

LT LEVEL	Foundational	Progressing	Proficient	Advanced
LT-LEVEL DESCRIPTION	Can name and record fractions for the parts of a whole or a set, such as in an area model and order them.	Can identify sets of equivalent fractions and compare fractions with like and unlike denominators.	Can identify and create unique concrete, pictorial, and symbolic equivalent fractions.	Can recognize and apply a patterns in numbers to divide and multiply to make equivalent fractions.
CRITICAL FEATURES / DISCERNMENTS	Fractions represent parts of a whole number. Recognizes what the numerator and denominator represent in a fraction.	Different fractions can equal the same amount.	Fractions can be represented and thus interpreted through various forms.	Equivalent fractions can be made by either multiplying or dividing the numerator and denominator by the same amount, as long as they remain whole numbers.
SUCCESS CRITERIA	Students can identify what fraction of a whole is being represented.	Students can compare and determine which fractions are equivalent.	Students can use various manipulatives and methods to create and express equivalent fractions.	Students can use multiplication and division to discover equivalent fractions both simplifying and creating higher terms for the fraction.
PROCESS(ES)	C, CN, ME, R, V	C, CN,PS, R, V	C, CN,PS, R, V	C, CN,PS, R, V
PROFICIENCY	Developing Conceptual	Conceptual & Adaptive Reasoning	Conceptual & Strategic Competence	Conceptual & Procedural & Adaptive Reasoning & Strategic Competence
CURRICULUM EXPECTATION	Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators? [C, CN, PS, R, V]			

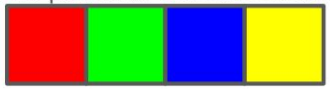
PERIMETER TRAINS PROBLEM - Sequencing related problems is hard.

Growth patterns via perimeter (concept development, guided/structured inquiry)

This shape has a perimeter of 4 units.

What is the perimeter of this shape?

What is the perimeter of a train containing 4 squares?



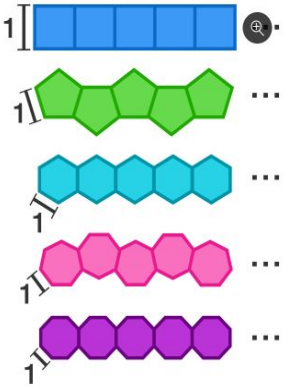
14 squares? 40 squares? 400 squares? etc.

We could put the problem in a story or meaningful context eg. seating for a party or building a walkway (and putting a flower border).

Think about how the SEQUENCING of the problems develops.

Excellent teachers are careful about their selection and sequencing of problems.

Perimeter growth pattern (rich task concept elaboration through practice)

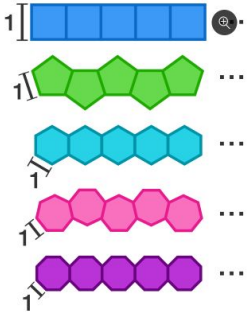


Juli builds several shape trains using different shapes. Each shape has all sides equal to 1 unit. Which of the trains cannot have a perimeter of 50 units?

For any shape trains which can have a perimeter of 50 units, how many copies of each shape will be needed?

[Could we put this in the context of a story/situation?]

Perimeter growth pattern (problem solving, exploration, problem-posing,)



Are there any other regular shapes with all sides equal (besides a 50 sided shape) that could be used in the same way to create a shape train with a perimeter of 50 units? How many different ones are there?

What if we chose to use combinations of different shapes? (eg. square-pentagon-square-pentagon) Are there other ways to get a perimeter of 50 using no more than 2 different shapes? 3 different shapes?

Open Inquiry/Exploration
Problem-posing

Concept Elaboration & Practice
Problem Solving w Rich Tasks

Concept Exploration & Development
Guided/Structured Inquiry

That different tasks serve different purposes is not obvious to novices - needs to be discerned.

Productive Struggle (Warshauer, 2015).

- Self-regulatory persistence and effort in making sense of a problematic learning situation.
- Remain intellectually challenged.
- Maintain the belief that one is capable of meeting the challenge.

Related concepts:
Resilience, Persistence,
Productive Disposition (Kilpatrick et al.)
Growth Mindset (Dweck, Boaler)
Model refinement (Papert)
Desirable difficulties (Bjork & Bjork)
Productive Failure (Kapur)
Deep Practice (Coyle)

Finger counting problem

Starting at Thumb (1) count each finger to Pinky (5).
Then count back to Thumb (don’t double count pinky or thumb when changing direction).
What finger are you on when you say ‘1000’?

Anticipating Levels of Skilled Performance is hard

Brute force non-patterned approacher	Pattern Noticer	Pattern Describer	Pattern Explainer	Math Object Creator
Uses a brute force approach but does not notice or use any patterns.	Groups or notices one or more patterns and uses diagrams to describe the pattern visually.	Notices patterns and is able to describe the pattern in words eg. "Starts at 1 and increased by 8 each time. " "The pattern of fingers repeats every 8 numbers. We start over on the thumb after every 8 counts."	Able to explain that since the pattern repeats every 8 counts and 1000 is evenly divisible by 8 then it is the finger that contains all the multiples of 8, which is the index finger. [Modulo 8 / Clock arithmetic argument.]	Creates a novel / unexpected mathematical artifact to assist in representing and communicating a correct solution to the problem. May be able to use to solve additional problems.

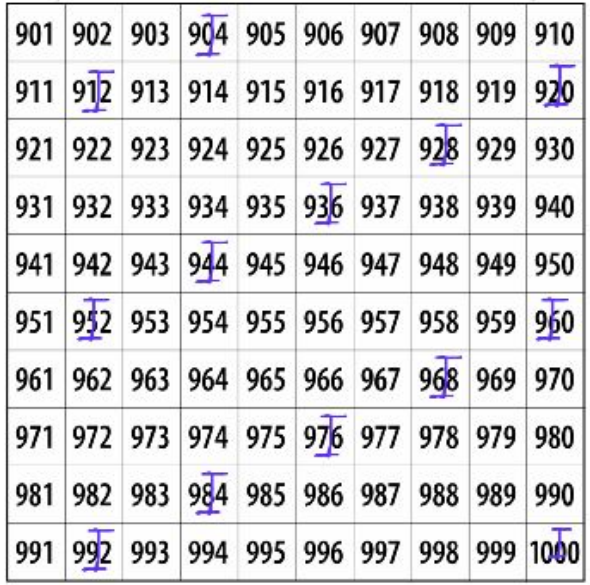
Recognizing the different levels of skilled approaches to a task/problem is hard. This must be discerned and requires ++ xx >>> experience & exposures. Simultaneously discerning that these levels in this type of learning situation are value-laden / weighted and not necessarily 'developmental', i.e. more context/purpose dependent.
Levels of skilled performance are somewhat 'analogous' to different ways of knowing→ in some situations some ways of knowing are preferred/valued.

Anticipated Responses - Exhaustive Techniques - Checks every case. No patterns noticed.

On hand or diagram of hand.	Counts every finger and keeps record of every number. No attempt to find a pattern.	
On thousands chart	Colours chart by fingers all the way to 1000. May not see patterns.	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

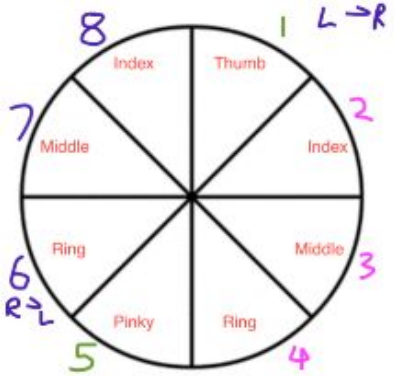
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200



901	902	903	904	905	906	907	908	909	910
911	912	913	914	915	916	917	918	919	920
921	922	923	924	925	926	927	928	929	930
931	932	933	934	935	936	937	938	939	940
941	942	943	944	945	946	947	948	949	950
951	952	953	954	955	956	957	958	959	960
961	962	963	964	965	966	967	968	969	970
971	972	973	974	975	976	977	978	979	980
981	982	983	984	985	986	987	988	989	990
991	992	993	994	995	996	997	998	999	1000

Anticipated Responses - Mathematical Object Creator.

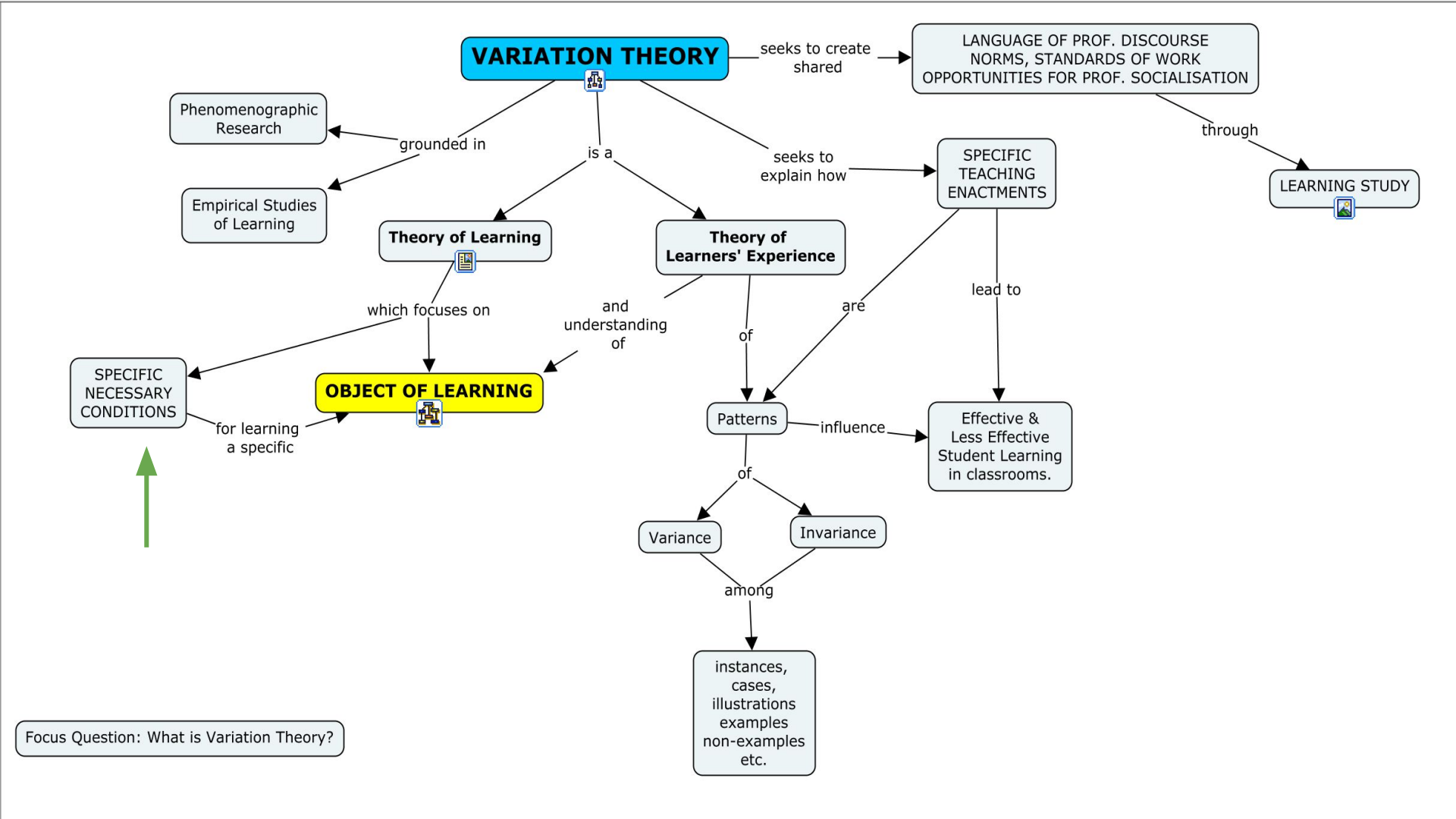
Group creates a new mathematical object to assist with communicating their ideas eg. 8 finger hand, 8 hour clock, 8 column thousands chart, 8 cell cycle. Able to explain why and how they used it.



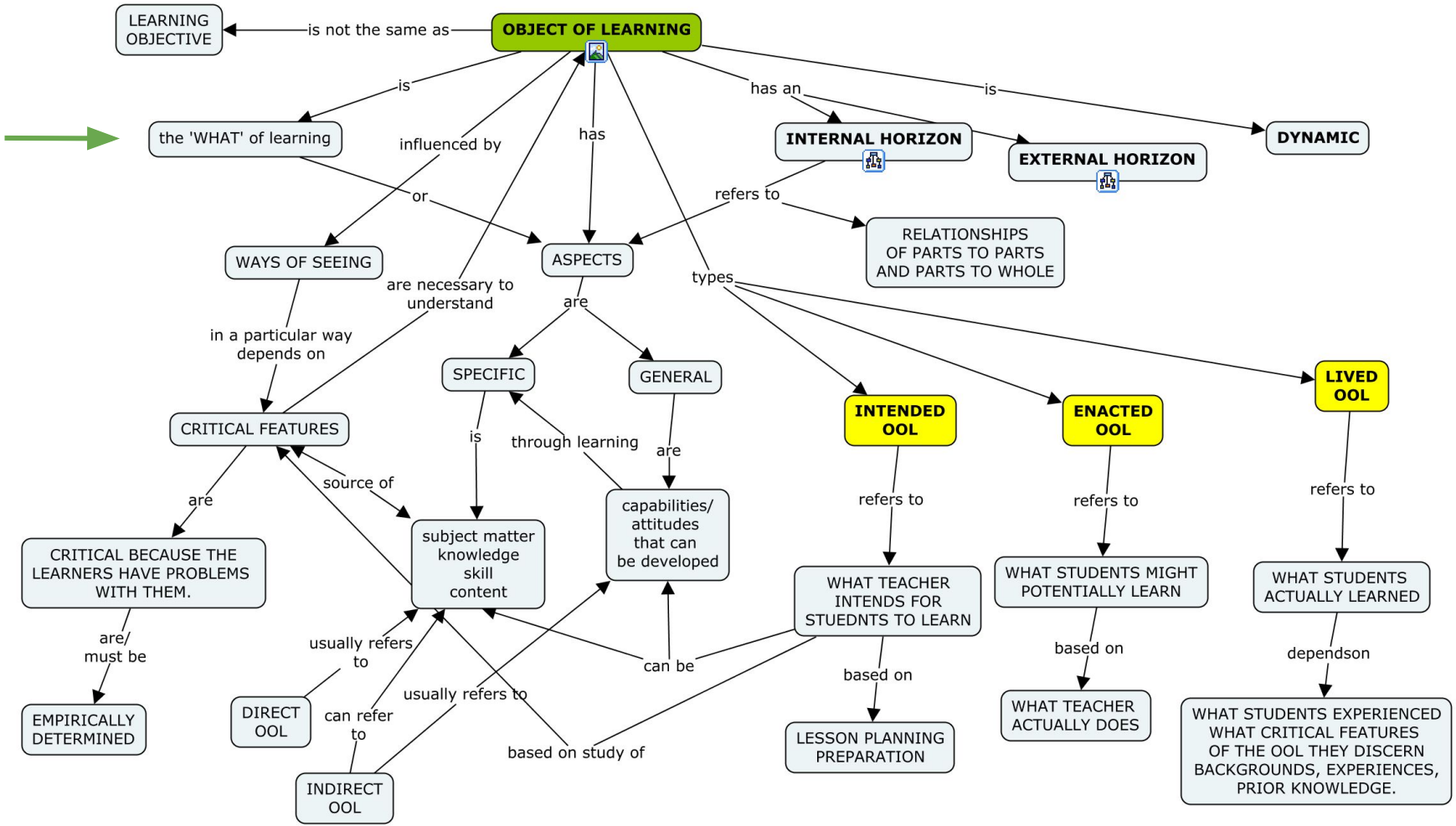
THUMB	INDEX	MIDDLE	RING
INDEX	MIDDLE	RING	PINKY

THUMB	INDEX	MIDDLE	RING	PINKY	RING	MIDDLE	INDEX
1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64
65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88
89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104
105	106	107	108	109	110	111	112
113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128
129	130	131	132	133	134	135	136
137	138	139	140	141	142	143	144
145	146	147	148	149	150	151	152
153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168
169	170	171	172	173	174	175	176
177	178	179	180	181	182	183	184
185	186	187	188	189	190	191	192
193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208

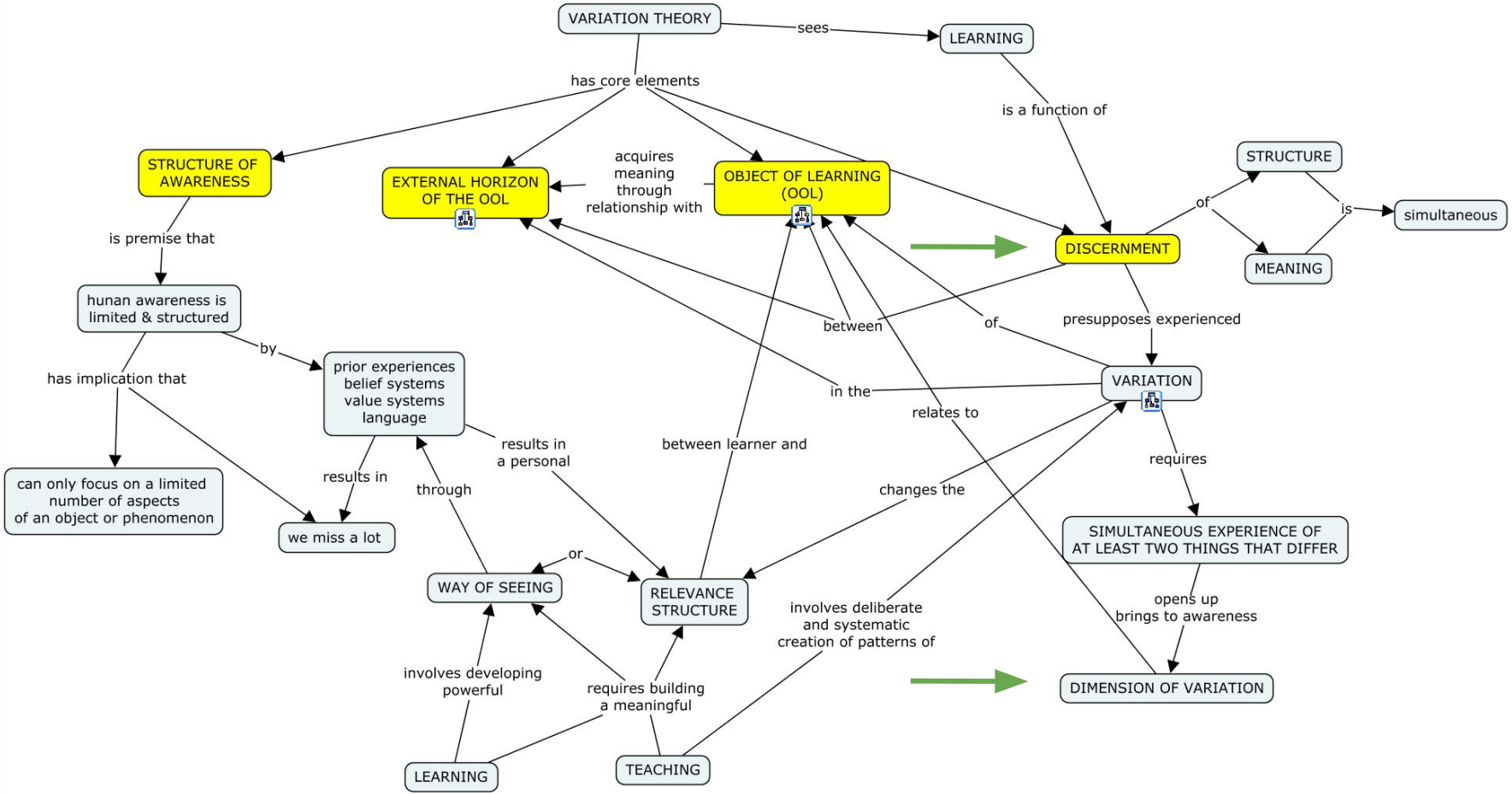
897	898	899	900	901	902	903	904
905	906	907	908	909	910	911	912
913	914	915	916	917	918	919	920
921	922	923	924	925	926	927	928
929	930	931	932	933	934	935	936
937	938	939	940	941	942	943	944
945	946	947	948	949	950	951	952
953	954	955	956	957	958	959	960
961	962	963	964	965	966	967	968
969	970	971	972	973	974	975	976
977	978	979	980	981	982	983	984
985	986	987	988	989	990	991	992
993	994	995	996	997	998	999	1000
1001	1002	1003	1004	1005	1006	1007	1008


VARIATION THEORY
seeks to create shared
LANGUAGE OF PROF. DISCOURSE
NORMS, STANDARDS OF WORK
OPPORTUNITIES FOR PROF. SOCIALISATION
through
LEARNING STUDY
Phenomenographic Research
grounded in
Empirical Studies of Learning
is a
Theory of Learning
Theory of Learners' Experience
seeks to explain how
SPECIFIC TEACHING ENACTMENTS
which focuses on
and understanding of
of
are
lead to
SPECIFIC NECESSARY CONDITIONS
for learning a specific
OBJECT OF LEARNING
Patterns
influence
Effective & Less Effective Student Learning in classrooms.
of
Variance
Invariance
among
instances, cases, illustrations examples non-examples etc.
Focus Question: What is Variation Theory?

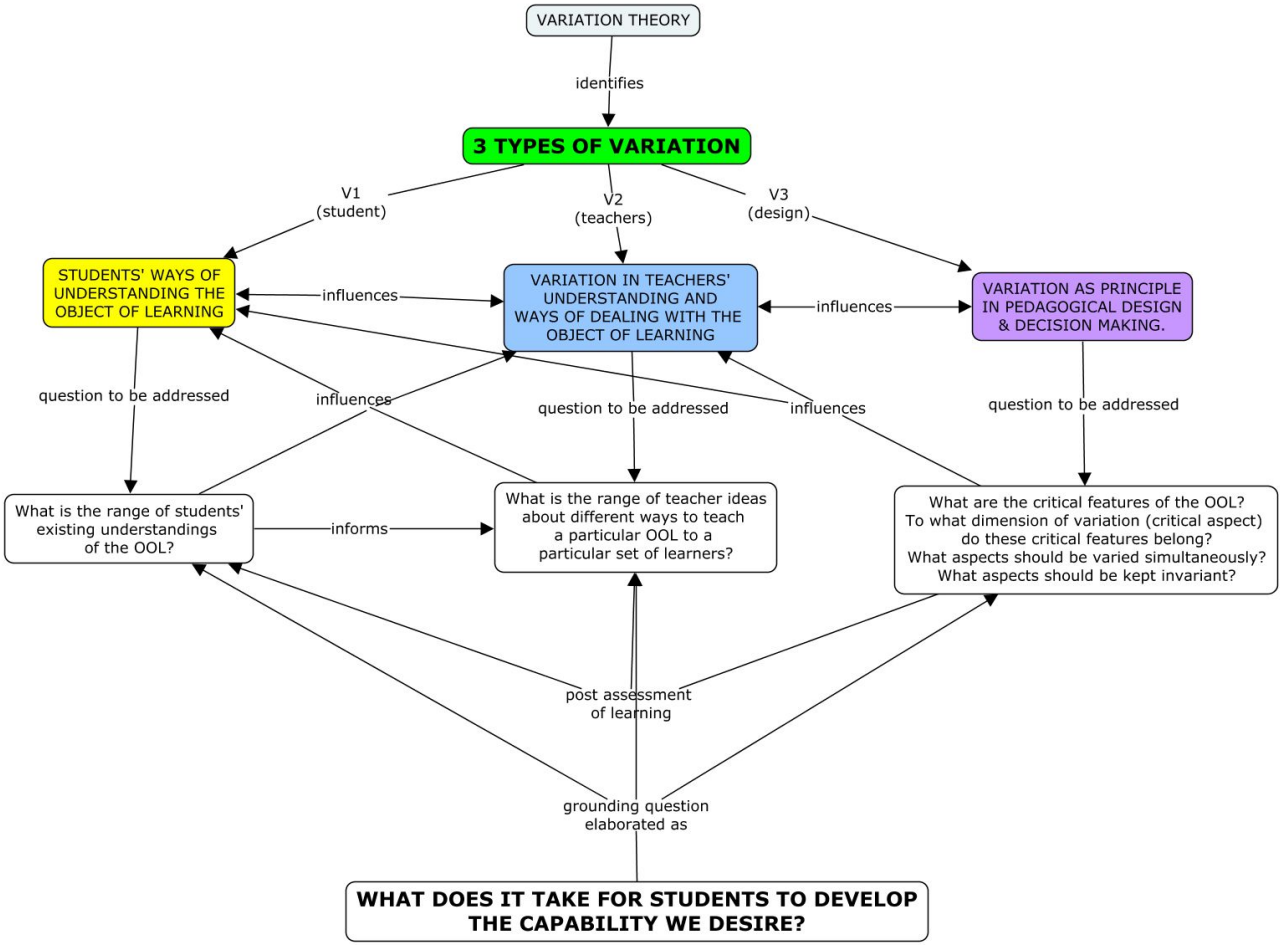
Focus Question: What is the Object of Learning (OOL) as conceived in Variation Theory?

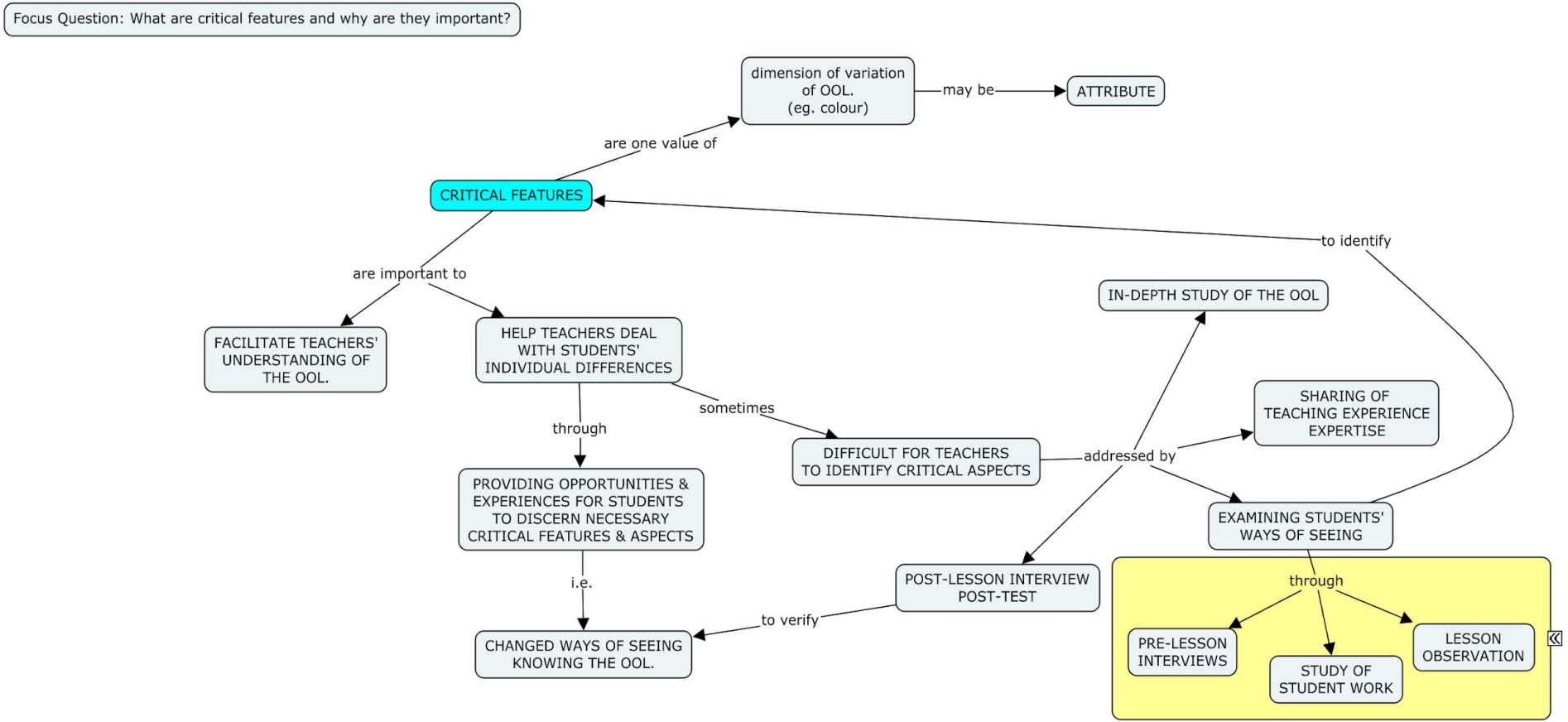


Focus Question: What are the elements of Variation Theory that have a significant influence on teaching and learning?



Focus Question: What are the different types of Variation in Variation Theory?




Focus Question: What are critical features and why are they important?
dimension of variation of OOL. (eg. colour)
may be
ATTRIBUTE
are one value of
CRITICAL FEATURES
to identify
are important to
FACILITATE TEACHERS' UNDERSTANDING OF THE OOL.
HELP TEACHERS DEAL WITH STUDENTS' INDIVIDUAL DIFFERENCES
IN-DEPTH STUDY OF THE OOL
through
sometimes
PROVIDING OPPORTUNITIES & EXPERIENCES FOR STUDENTS TO DISCERN NECESSARY CRITICAL FEATURES & ASPECTS
DIFFICULT FOR TEACHERS TO IDENTIFY CRITICAL ASPECTS
addressed by
SHARING OF TEACHING EXPERIENCE EXPERTISE
EXAMINING STUDENTS' WAYS OF SEEING
i.e.
POST-LESSON INTERVIEW POST-TEST
through
to verify
CHANGED WAYS OF SEEING KNOWING THE OOL.
PRE-LESSON INTERVIEWS
STUDY OF STUDENT WORK
LESSON OBSERVATION

An example

We ask a 6-year old-child: “How many fingers do you have on your left hand?”
She says: “Five.” Then you ask: “How many fingers do you have on your right hand?”
And she says: “Ten.”

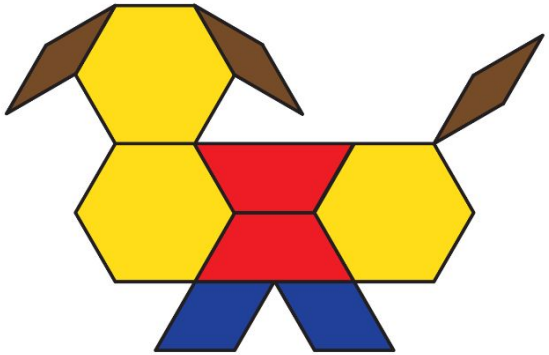
Do “five” and “ten” mean the same to her as they mean to us?

What is the child attending to? What isn’t the child attending to?

Teacher Knowledge: Relevant necessary aspects of Number & Counting
ordinal property - number names refer to a place in a stable order/sequence.
cardinal property - each number refers to a certain “manyness”
de/composability - numbers are wholes that can be **divided into parts** (part-whole relationship)**.**

Variation theory gives an insight into what has not been discerned.
A Learning Trajectory will suggest tasks to build the cardinal property and ability to compose and decompose numbers.
Games will help build fluency through productive struggle.

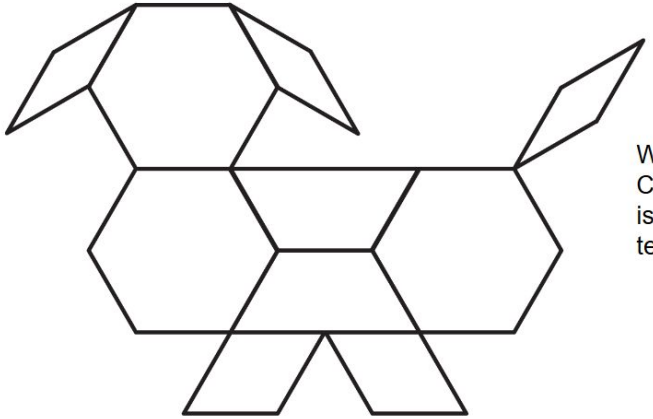
Reproduce this ~~pattern~~ design



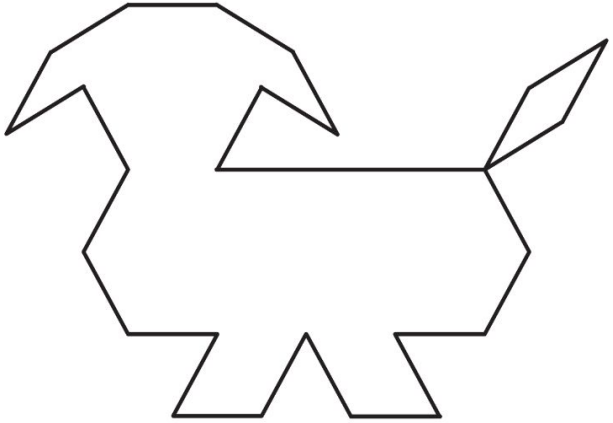
What is it possible for the learner to discern?

What questions can you ask?

What might be some difficulties learners might have?



WHAT HAS CHANGED/VARIED? What is it now possible for the teacher to discern?



Can you cover this picture using pattern blocks?

What is it possible for the learner to discern?

What is it possible for the teacher to discern about the learner?

Is there another way to cover the picture using the same blocks?Can you cover it with different blocks?

How many different ways can you cover it? Are there any blocks that don't change (are invariant)?

What is the most/least number of blocks that can be used? How do you know it is the most/least?

How many blocks did you use? Can you increase the number of blocks used by ___?

Can you cover it with any number of blocks between the least and most?

How many different types of blocks can you use?

What fraction/percentage of the blocks are____? (colour, shape, set.)

Variation Theory

→ Introduces idea of Critical Features.

→ Helps students become aware of Dimensions that can Vary.

→ Used to help them discern:

- Variations in task purposes/learning goals
- Variations in instructional strategies.
- Way mathematical concepts are organized/attended to by the Professional Discourse Community.
- Helps to identify that there can be variation in skilled performance.

Learning Trajectories

→ Introduces idea that learning is developmental and facilitated by meaningful tasks effectively facilitated.

→ Gives a framework for mathematical concept development (learn complex subject matter)

→ Helps in identifying LEVELS OF SKILLED PERFORMANCE.

- Suggests relevant tasks to help move from one level to another.

Game Mechanics

→ Are one way to structure and sequence learning experiences for productive struggle.

→ Productive struggle does not have to be isolated/individual struggle.

→ Systems are structured but can be hacked/gamed - designed to be 'played'.

Productive Struggle

→ Introduces idea that effort & persistence are required to learn complex subject matter.
(learn complex subject matter)

Practice

Identify some discrete area/topic/concept/skill that is difficult for learners in your course.
What do you believe they struggle what?
What is it they need to discern? How can they discern this (what needs to vary?)
What is the sequence of discernments or levels of skills?
What tasks are associated with which discernments/skills?
Develop a provisional learning trajectory - goals (discernments), path (sequence) and linked tasks.
Which if any parts might be amenable to game-based learning?

Questions & Follow-up

Please send comments on the session and questions to skkhan@ualberta.ca

References

Devlin, Keith. (2011). *Mathematics education for a new era. Video-games in the classroom.* CRC Press.

Clements, D. H. & Sarama, J. (2010). Learning Trajectories in Early Mathematics – Sequences of Acquisition and Teaching. In: Tremblay RE, Boivin M, Peters RDeV, eds. Bisanz J, topic ed. *Encyclopedia on Early Childhood Development*[online]. http://www.child-encyclopedia.com/numeracy/according-experts/learning-trajectories-early-mathematics-sequences-acquisition-and. Published July 2010. Accessed February 25, 2018.

Martin, F. (2014). *Necessary conditions of learning*. Routledge.

Warshauer, H.K. (2015). Productive struggle in Middle School Mathematics Classrooms. *Journal of Mathematics Teacher Education*. 18: 375. https://doi.org/10.1007/s10857-014-9286-3