Hydrodynamic Stability Theory (Mathematics 655)

LECTURER: Professor G. E. Swaters, Department of Mathematical & Statistical Sciences, CAB 537, Phone 780-492-7159.

Email: gswaters “@” ualberta “dot” ca

OTHER CONTACT INFORMATION: See my URL at http://pacific.math.ualberta.ca/gordon/.

PREQUISTES:

Introductory knowledge of Fluid Dynamics, Knowledge of solution techniques in ordinary and partial differential equations, or consent of instructor.

TEXTBOOK: Hydrodynamic Stability *by* P. G. Drazin & R. H. Reid, Cambridge University Press.

LECTURE TIME & LOCATION: TBD.

SECTION URL: http://pacific.math.ualberta.ca/gordon/teaching/instability.html

GENERAL COURSE DESCRIPTION: The principal purpose of this course is to present five aspects of the theory of hydrodynamic stability that have wide application in applied mathematics, engineering sciences, physics, meteorology and oceanography:

DETAILED DESCRIPTION:

Introduction to concepts in hydrodynamic stability theory: Linearization and the method of normal modes. Liapunov and spectral stability. Asymptotic, neutral and marginal stability. Relationship between normal-mode stability and the Laplace Transform.

Rayleigh-Benard Instability/Convection: Thermal instability of a compressible fluid heated from below under the influence of gravity. Rigorous derivation of the Boussinesq stability equations and boundary conditions. Stability as a function of the Prandtl and Rayleigh numbers. Exchange of stabilities. The role of rotation in Rayleigh-Benard convection.

Rayleigh Instability: Classical inertial instability of a parallel shear flow of a homogeneous fluid. Squire’s Theorem. Derivation of Rayleigh’s Stability Equation. Rayleigh’s and Fjortoft’s Stability Theorems. Howard’s Semi-Circle Theorem. Vortex-sheet and non-inflectional shear layer models. Critical layers associated with neutral modes. Transverse structure of the perturbation Reynolds stress for neutral modes. Introduction to the application of noncanonical infinite-dimensional Hamiltonian dynamics in the general stability theory for 2D flows.

Kelvin-Helmholtz Instability: Inertial instability of a parallel shear flow of a density-stratified fluid under the influence of gravity. Internal gravity waves as neutral disturbances in stratified shear flows. Derivation of the Taylor-Goldstein equation. Vortex-sheet and non-inflectional shear layer models. Generalization of Howard's Semi-Circle Theorem. Synge's Stability Theorem (the generalization of Rayleigh's Theorem). Miles (Richardson Number) Theorem.

Weakly-nonlinear evolution of marginally unstable flows: Introduction to the development of the asymptotic theory describing the finite-amplitude evolution of normal mode perturbations in slightly super or subcritical flows - including mean flow modification. Derivation of the Landau equation for the normal mode amplitude. Bounded and explosively unstable solutions of the Landau equation. Derivation of the nonlinear Klein-Gordon (*KG*) equation for the time-only evolution of the normal mode amplitude. Solution of the *KG* equation in terms of Jacobian elliptic functions. Derivation of a generalized nonlinear Schrödinger equation (*NLS*) describing the space-time finite-amplitude development of neutrally stable normal modes. Stokes wave solution to *NLS* and modulational or Benjamin-Feir instability.

GRADING: There are no examinations. The final grade will be determined by the students' performance based on assignments. An overall course mark of 50% or more guarantees a passing grade of at least C-. An overall course mark of 90% or more guarantees a grade of at least A.

ACADEMIC INTEGRITY: The University of Alberta is committed to the highest standards of academic integrity and honesty. Students are expected to be familiar with these standards regarding academic honesty and to uphold the policies of the University in this respect. Students are particularly urged to familiarize themselves with the provisions of the Code of Student Behaviour (online at www.governance.ualberta.ca) and avoid any behaviour which could potentially result in suspicions of cheating, plagiarism, misrepresentation of facts and/or participation in an offence. Academic dishonesty is a serious offence and can result in suspension or expulsion from the University.