This test is designed for students enrolled in one of Math 134, 144, or 154 at the University of Alberta. It is a test of the skills needed to successfully complete a university level calculus course. The results of this test are to provide you with information for your self-assessment only. If the results of this test suggest some weakness in your skills (you achieve fewer than 15 correct answers out of 25), you are advised to take some from of remedial action. The Support Centre for Mathematical and Statistical Sciences, located in room 528 of the Central Academic Building (CAB), has resources that can help. Go to this link to view the solutions.

This set of questions is designed to take 60 minutes and to be written without a calculator. Keep in mind that the purpose of this test is advisory, and that a correct guess at an answer will only hide a weakness. If you do not know a correct answer, leave it blank.

1. The solutions to the equation \( x^2 + 6x + 8 = 24 \) are:
   (a) \(-6, 6\)  (b) \(-2, 8\)  (c) \(-8, 2\)  (d) \(2, 6\)  (e) \(-8, 8\)

2. Which of the following expressions are equivalent when \( x > 1 \) and \( x \neq y \)?
   
   I. \( \frac{x - 6}{x - x^2}, \frac{6}{x^3} \)
   
   II. \( \frac{-3}{8}, \frac{11 - 3x}{11 + 8x} \)
   
   III. \( \frac{x}{4}, \frac{x^2 - xy}{x - 4y} \)
   
   (a) I  (b) II  (c) III  (d) I and II  (e) I, II and III

3. If \( x > 0 \), then the expression \( \sqrt{x^3} + \sqrt{1 + \frac{1}{\sqrt{x}}} \) is equivalent to:
   
   (a) \( x^{\frac{3}{2}} + |1 + x^{\frac{1}{3}}|^{\frac{1}{2}} \)
   
   (b) \( x^{\frac{3}{2}} + 1 + x^{\frac{1}{6}} \)
   
   (c) \( x^{\frac{3}{2}} + (1 + x^{\frac{1}{3}})^{\frac{1}{2}} \)
   
   (d) \( x^{\frac{3}{2}} + (1 + x^{\frac{1}{3}})^{\frac{1}{4}} \)
   
   (e) none of these

4. The solution of the inequality \( \frac{1}{2x} + \frac{1}{x} > 4 \) is:
   
   (a) \( x < \frac{3}{8} \)
   
   (b) \( 0 < x < \frac{3}{8} \)
   
   (c) \( x > \frac{3}{8} \)
   
   (d) \( 0 < x < \frac{1}{12} \)
   
   (e) \( x < \frac{1}{12} \)

5. Simplify \( \frac{2}{\sqrt{x} + 3} \) by rationalizing the denominator.
   
   (a) \( \frac{4}{x + 9} \)
   
   (b) \( \frac{2\sqrt{x} - 6}{x - 3} \)
   
   (c) \( \frac{2\sqrt{x} + 6}{x - 9} \)
   
   (d) \( \frac{2\sqrt{x} - 6}{x - 9} \)
   
   (e) none of these

6. If \( x = 2 \), then \( |x^2 - 16| \) is equivalent to:
   
   (a) \( -(x^2 - 16) \)
   
   (b) \( x^2 - 16 \)
   
   (c) \( x^2 + 16 \)
   
   (d) undefined
   
   (e) none of these
7. The expression \( \frac{1}{x^2} - \frac{1}{y^2} \) is equivalent to:

(a) \( \frac{1}{x+y} \) (b) \( x+y \) (c) \( \frac{1}{xy(x+y)} \) (d) \( \frac{1}{xy(x+y)} \) (e) \( -\frac{1}{xy(x-y)} \)

8. The expression \( \frac{1}{2x^2} + \frac{1}{9x^6} - \frac{3}{x^6} \) can be written as:

(a) \( 2x^2 + x^{-5/6} - 3x^6 \) (b) \( 2x^2 + x^{-5/6} + 3x^6 \) (c) \( \frac{1}{2}x^2 + x^{-6/5} - 3x^6 \)

(d) \( \frac{1}{2}x^2 + x^{-6/5} - 3x^6 \) (e) none of these

9. If \( f(x) = 2x^2 + 1 \), then \( \frac{f(x+h) - f(x)}{h} \) is equivalent to:

(a) \( \frac{2+h}{h} \) (b) 1 (c) \( 4x \) (d) \( 4x + 2h \) (e) \( 4x + 2h + 2 \)

10. If \( f(x) = \sqrt{x} \) and \( g(x) = x^2 + 4 \) then \( f(g(x)) \) is equivalent to:

(a) \( \sqrt{x} (x^2 + 4) \) (b) \( x + 4 \) (c) \( x + 2 \) (d) \( \sqrt{x^2 + 4} \) (e) none of these

11. The domain of \( f(x) = \frac{x-4}{\sqrt{x^2 + x - 6}} \) is:

(a) \(-3 < x < 2\) (b) \( x < -3 \), \( x > 2 \) (c) \( -3 \leq x \leq 2 \) (d) all real numbers except \( x = 2 \) and \( x = -3 \)

(e) all real numbers except \( x = 2 \), \( x = -3 \) and \( x = 4 \)

12. The expression \( 64a^3 - b^3 \) can be factored as:

(a) \( (4a-b)^3 \) (b) \( (4a+b)(16a^2 + 4ab + b^2) \) (c) \( (4a-b)(16a^2 - 4ab - b^2) \) (d) does not factor

(e) none of these

13. The expression \( 2(x+1)(x-1)^{-1} - (x+1)^2(x-1)^{-2} + 3(x+1)(x-1)^{-2} \) can be factored as:

(a) \( (x+1)^2(x-1)^2 \) (b) \( x(x+1)(x-1)^{-2} \) (c) \( 2(x+1)(x+2)(x-1)^{-2} \) (d) \( 4x(x+1) \)

(e) none of these
Answer questions 14 and 15 for the function given by:

\[ f(x) = \begin{cases} 
-1, & x < -1 \\
x^2, & -1 \leq x < 2 \\
-2x+7, & x \geq 2 
\end{cases} \]

14. The value of \( f(-1) \) is:
   (a) -1   (b) 0   (c) 1   (d) 9   (e) not defined

15. The domain of \( f \) is:
   (a) \(-1 \leq x < 2\)   (b) all real numbers   (c) \((-\infty, 2) \cup (2, \infty)\)   (d) \(x < -1\)   (e) none of these

16. The vertex of the parabola given by \( y = x^2 + 12x + 16 \) is at the point:
   (a) \((-3, -2)\)   (b) \((-3, 1)\)   (c) \((-3, 7)\)   (d) \((3, 1)\)   (e) none of these

17. The root(s) of the equation \( y = x^{\frac{2}{3}} - 2x^{\frac{8}{3}} \) is/are:
   (a) 0   (b) 0, 2   (c) 0, \(\sqrt[3]{2}, -\sqrt[3]{2}\)   (d) \(\sqrt[3]{2}, -\sqrt[3]{2}\)   (e) none of these

18. An equation of the line passing through the point \((2, -3)\) and perpendicular to the line \(x - 3y = 6\) is:
   (a) \(y - 3 = 3(x + 2)\)   (b) \(y + 3 = 3(x + 2)\)   (c) \(y + 3 = \frac{1}{3}(x - 2)\)   (d) \(y - 3 = -\frac{1}{3}(x + 2)\)   (e) none of these

19. The quadratic function that satisfies \( f(4) = 0 \), \( f(3) = 0 \) and \( f(2) = 1 \) is:
   (a) \(f(x) = (x - 4)(x - 3)\)   (b) \(f(x) = (x + 4)(x + 3)\)   (c) \(f(x) = \frac{1}{2}(x - 4)(x - 3)\)   (d) \(f(x) = \frac{1}{4}(x + 4)(x + 3)\)   (e) none of these

20. Jay wishes to go from his tent to the ice cream stand as pictured below. The distances between X and Y is 4 Km, between X and Z is 7 Km, and between A and Z is 4 Km. He will jog at 8 Km/hr to his canoe at the dock and then paddle directly to the ice cream stand at 10 Km/hr. How long will it take Jay to get his ice cream treat? (Assume continuous motion (no stops) and no current in the river.)

   (a) 1 hr   (b) \(\frac{11}{8}\) hr   (c) \(\frac{9}{10}\) hr   (d) \(\frac{1}{2} \cdot \frac{\sqrt{55}}{10}\) hr   (e) none of these
21. The expression \((\sin x + \cos x)^2\) is equivalent to:

(a) 1    (b) 1 + 2\sin x \cos x    (c) \sin(x^2) + \cos(x^2)    (d) \sin(x^2) + 2\sin x \cos x + \cos(x^2)

(e) none of these

22. Given that \(\sin(\theta) = \frac{12}{13}\) and \(0 < \theta < \frac{\pi}{2}\), the value of \(\sin(2\theta)\) is:

(a) \(\frac{5}{13}\)    (b) \(\frac{6}{13}\)    (c) \(\frac{24}{13}\)    (d) \(\frac{25}{169}\)    (e) \(\frac{120}{169}\)

23. On its domain, \(\frac{\csc \theta \tan^2 \theta}{\sec^3 \theta}\) is equivalent to:

(a) \(\sin \theta \cos \theta\)    (b) \(\sin(\cos \theta)\)    (c) \(\sin^2 \theta\)    (d) \(\sin^2 \theta \tan^3 \theta\)    (e) none of these

24. For what values of \(t\), \(0 \leq t \leq 2\pi\), does \((2\sin t - 1)(\cos t + 1) = 0\)?

(a) \(0, \frac{\pi}{3}, \frac{2\pi}{3}\)    (b) \(\pi, \frac{\pi}{6}, \frac{5\pi}{6}\)    (c) \(0, \pm \frac{\pi}{3}, \pm \frac{2\pi}{3}\)    (d) \(\pi, \pm \frac{\pi}{3}, \pm \frac{2\pi}{3}\)    (e) none of these

25. A man walks away from a spotlight that is located at the top of a 10-foot pole. If the man is six feet tall, and the light casts his shadow upon the ground, how long is his shadow when he is 4 feet from the base of the spotlight?

(a) 6 feet    (b) 4 feet    (c) 5 feet    (d) \(\frac{12}{5}\) feet    (e) none of these